Eur. Phys. J. B **4**, 139–141 (1998)

THE EUROPEAN PHYSICAL JOURNAL B

Rapid Note

How the financial crash of October 1997 could have been predicted

N. Vandewalle[1,a], M. Ausloos[1], Ph. Boveroux[2], and A. Minguet[2]

[1] SUPRAS, Institut de Physique B5, Université de Liège, 4000 Liège, Belgium
[2] Théorie monétaire et finances B31, Faculté d'Économie, Gestion et Sciences Sociales, Université de Liège, 4000 Liège, Belgium

Received: 23 March 1998 / Revised and Accepted: 23 April 1998

Abstract. From the analysis of (closing value) stock market index like the Dow Jones Industrial average and the S&P500 it is possible to observe the precursor of a so-called crash. This is shown on the Oct. 1987 and Oct. 1997 cases. The data analysis indicates that the index divergence has followed twice a "universal" behavior, *i.e.* a logarithmic dependence, superposed on a well defined oscillation pattern. The prediction of the crash date is remarkable and can be done two months in advance. In the spirit of phase transition phenomena, the economic index is said to be analogous to a signal signature found in a two dimensional fluid of vortices.

PACS. 01.75.+m Science and society – 89.90.+n Other areas of general interest to physicists

Even though a stock market crash is considered as a highly unpredictable event, it should be noted that it takes place systematically during a period of generalized euphory. Are we able to quantify that euphory? Are we able to observe precursors of a crash? In the following, we will concentrate on the numerical aspect of crash precursor analysis, leaving more psychological and economical questions aside. In August 1997, we performed a series of investigations in order to emphasize crash precursors. We have used daily data of stock markets like the Dow Jones Industrial Average (DJIA) and the Standard & Poor 500 (S&P500). A strong indication of a crash event or a rupture point in between the end of October 1997 to mid-November 1997 was numerically discovered [1]. This results from an analysis of the similarities between two long periods: 1980-87 and 1990-97. For the first period, the analysis was performed on data ending two months before the so-called Black Monday, *i.e.* October 19, 1987. For the second period, the data was considered till August 20th, 1997. These two sets of data are drawn in Figures 1a-b in the case of the Dow Jones. In both cases, we should note that an anomalous increase of the index exists up to two months before the crash. The present report supports the idea that this increase is a precursor of the crash.

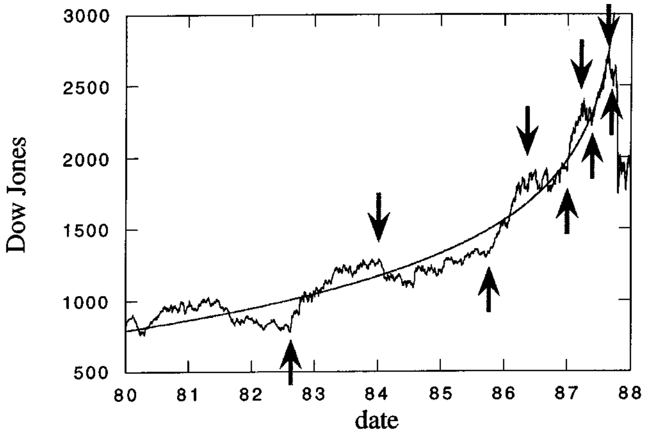
The application of statistical physics ideas to the forecasting of stock market behavior and crashes has been proposed earlier [2,3] following the pioneer work of physicits in economy [4–8]. It was proposed that an economic index $y(t)$ increases as a complex power law, *i.e.*

$$y = A + B\left(\frac{t_c - t}{t_c}\right)^{-m} \times \left[1 + C\cos\left(\omega \ln\left(\frac{t_c - t}{t_c}\right) + \phi\right)\right] \quad \text{for} \quad t < t_c \tag{1}$$

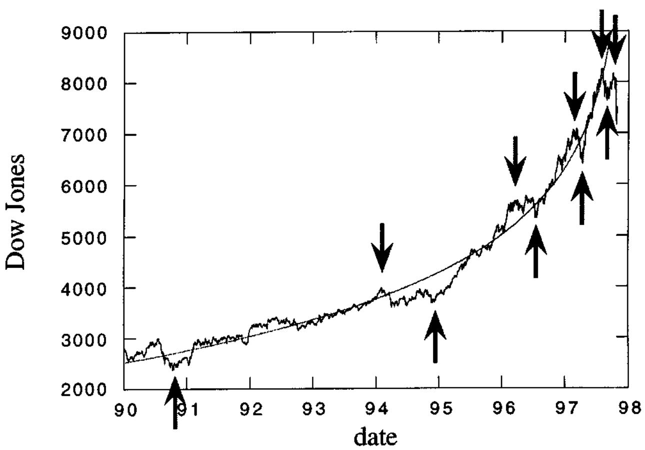
where t_c is the crash-time or rupture point, A, B, m, C, ω and ϕ are free parameters. This evolution $y(t)$ is in fact the real part of a power law divergence at $t = t_c$ with a complex exponent $m + i\omega$. The law for $y(t)$ diverges at $t = t_c$ with an exponent m while the period of the oscillations converges to the rupture point at $t = t_c$. This law is similar to that of critical points at so-called second order phase transitions [9], and generalizes the scaleless situation for cases in which discrete scale invariance [10] is presupposed. This relationship (1) was already proposed in order to fit experimental measurements of sound wave rate emissions prior to the rupture of heterogeneous composite stressed up to failure [11]. The same type of complex power law behavior has been observed as a precursor of the Kobe earthquake in Japan [12]. Such log-periodic corrections have been recently reported in biased diffusion on random lattices [13].

Fits using equation (1) were already performed on the S&P500 data [1,2] for the period preceding the 1987 October crash. It should be stressed that the numerical parameter values are not robust against small data perturbations. It is well known indeed that a nonlinear seven

[a] e-mail: `vandewal@gw.unipc.ulg.ac.be`



(a)



(b)

Fig. 1. The daily evolution of the Dow Jones Industrial Average: (a) from January 1980 to December 1988 and (b) from January 1990 to October 1997. The continuous curves are best fits using a logarithmic divergence, *i.e.* setting $C = 0$ in equation (2): (a) $A = -499.4 \pm 16.1$, $B = -532.9 \pm 5.6$, $t_c^{div} = 87.85 \pm 0.02$ and (b) $A = -1919.6 \pm 38$, $B = -1762 \pm 13.4$, $t_c^{div} = 97.92 \pm 0.02$. Downward (resp. upward) arrows denote successive maxima (resp. minima) of the log-periodic oscillations.

parameter fit is highly unstable from a numerical point of view [14]. Indeed, suppressing the contribution of the oscillations in equation (1), *i.e.* setting $C = 0$ and thus reducing the number of free parameters, implies that the best fit leads to an exponent $m = 0.7$ quite larger than that found in [2], *i.e.* $m = 0.33$ for $C \neq 0$. On the other hand, in reference [3], various values of m were in fact reported as ranging from 0.53 to 0.06 for various indices and events (upsurges and crashes).

However, there are strong physical arguments stipulating that m could be or even should be "universal" by analogy with second order phase transitions [15]. "Universality" in this case means that the value of m should be the same for any crash and for any index. In so doing a single model should describe the phase transition behavior while the exponent m would define the model and be the only parameter defining the "universality". One can find in the literature on phase transitions a limited set of exponent values and models appropriate to many given physical cases [15].

An interesting behavior to be considered is the logarithmic divergence, corresponding to the $m = 0$ limit. We propose that the "universal exponent m" is in fact close to zero, *i.e.* the divergence of the index y for t close to t_c should be

$$y = A + B \ln\left(\frac{t_c - t}{t_c}\right) \times \left[1 + C\cos\left(\omega \ln\left(\frac{t_c - t}{t_c}\right) + \phi\right)\right] \quad \text{for} \quad t < t_c. \tag{2}$$

This logarithmic behavior is known in physics as characterizing the specific heat (a "four point correlation function") of the $d = 2$ Ising model or the $d = 3$ XY-model [4,14,16]. Physically, it represents in the latter case a transformation from a disordered vortex fluid state with equal number of vortices with opposite vorticity to an ordered state with "particles" composed from a pair of vortices with different polarities. Such a behavior appears in systems governed by fluid-like contagions [17]. The contagion behavior from a market place to another is of course the key ingredient leading to a crash. The mean value of the order parameter is not defined over long-range scales, but a phase transition nevertheless exists because there is some ordered state on "small" scales. In addition to the neat physical interpretation of the relationship (2), the advantages are found in the facts that (i) the total number of parameters is reduced by one with respect to equation (1), and (ii) the log-divergence seems to be closer to reality than the power-divergence as we show below.

As in critical point data analysis the optimum test consists in separating the most diverging term from the others and next searching for the correction to scaling [18]. In order to test the validity of equation (2) in the vicinity of crashes, we did not fit the data using this 6-parameter function though. In fact, we have separated the problems of the divergence itself and the oscillation convergences on the other hand, in order to extract two values for the rupture point t_c: (i) t_c^{div} for the power (or logarithmic) divergence and (ii) t_c^{osc} for the oscillation convergence. In so doing, we examine on both the divergence of the trend and the convergence of the oscillations.

First, we fitted the two indices using the logarithmic divergence with only 3 free parameters, *i.e.* A, B and t_c^{div}. The results of the fits to equations (1, 2) are summarized in Table 1. A second rupture point t_c^{osc} was estimated by selecting the successive maxima and the minima of the oscillations (see the arrows in Figs. 1a-b). Due to the log-periodicity in equation (2), the following relation

$$\frac{t_{n+1} - t_n}{t_n - t_{n-1}} = \frac{1}{\lambda} \tag{3}$$

Table 1. Fundamental parameters of equations (1-4) found for the DJIA and S&P500 indices for both 1980-87 and 1990-97 periods. Time is expressed in years. The notations for t_c are such that *e.g.* 97.90 means the calendar date corresponding to the 90-th day of a 100-day year in 1997. The number of open days per year on Wall Street is about 261 days, the exact value depending on the number of holidays falling on week ends. Two values of t_c^{div} correspond to respectively a fit using a logarithmic divergence ($m = 0$) and a fit using a power law divergence ($m \neq 0$). The true date of the October 1987 crash in the above units gives $t_c = 87.79$ and for the October 1997 crash is $t_c = 97.81$, *i.e.* quasi the predicted dates.

index – (period)	$t_c^{div}(m = 0)$	$t_c^{div}(m \neq 0)$	λ	t_c^{osc}
DJIA (80-87)	87.85 ± 0.02	88.46 ± 0.04	2.382 ± 0.123	87.91 ± 0.10
DJIA (90-97)	97.92 ± 0.02	98.68 ± 0.04	2.278 ± 0.045	97.89 ± 0.06
S&P500 (80-87)	87.89 ± 0.03	88.78 ± 0.05	2.528 ± 0.127	87.88 ± 0.07
S&P500 (90-97)	97.90 ± 0.02	98.67 ± 0.04	2.549 ± 0.163	97.85 ± 0.08

holds true where $\lambda = \exp(\omega/2\pi)$ and t_{n-1}, t_n, t_{n+1} are the successive converging maxima (resp. minima). After estimating λ through equation (3), the rupture point t_c^{osc} is found from

$$t_c^{osc} = \frac{t_n - \frac{t_{n+1}}{\lambda}}{1 - \frac{1}{\lambda}} \,. \tag{4}$$

The λ and t_c^{osc} values obtained for the Dow and S&P500 are given in Table 1 for both 1980-87 and 1990-97 periods. It should be noted that the value of λ seems to be also universal in the sense that this rate of the convergence is always in the range 2.3−2.5. A discussion about common values of λ for natural phenomena can be found in [10].

Let us now compare both divergence and convergence laws. It should be stressed that the t_c^{osc} and t_c^{div} values are closer in the case of a logarithmic divergence ($m = 0$) than in the case of a power divergence ($m \neq 0$) of references [2,3]. Both rupture points for $m = 0$ are consistent with equation (2), *i.e.* $t_c^{osc} \approx t_c^{div}$. Moreover, the fits readily show that stock market indices follow a logarithmic law divergence. In so doing, the crash of 1987 was re-predicted within an error of 3 weeks. For the 90-97 period, our results suggested as early as August 1997 that a crash or a rupture point was highly probable between the end of October and mid-November 1997. It should be noted that t_c^{osc} and t_c^{div} are extreme dates since the index should fall before it reaches infinity. Taking into account this finiteness of the indices, our precursor analysis is thus quite relevant with respect to the crashes of 1987 and 1997. A continuous follow-up of the data taking into account the oscillations is thus a neat way of predicting a stock market rupture point.

Note added after this paper was completed. A preprint [19] by Laloux *et al.* discusses that it is hard to predict financial crashes, based on some lack of reliability of the data and its subsequent analysis according to various authors. One may argue that statistical data analysis techniques and results (like the nonlinear fits occurring in critical exponent studies) much depend on practical experience indeed. To our knowledge, no set of rigorous rules exists for finding the unique (thus best) solution. Moreover, data analysis requests many points, not always available for financial matters. About this debate, Stauffer has argued [20] that we will be dead before observing the 100th Wall Street crash, *i.e.* before getting enough data. Moreover, what is a crash [2]? Should we not rather discuss ruptures [1]? Then, what is a rupture? Beside mathematical rigor, we conclude/argue that we should use physical insight [21–24].

References

1. H. Dupuis, Trends Tendances **28** (32), 26 (september 1997) .
2. D. Sornette, A. Johansen, J.-P. Bouchaud, J. Phys. I France **6**, 167 (1996).
3. J.A. Feigenbaum, P.G.O. Freund, Int. J. Mod. Phys. B **10**, 3737 (1996).
4. B.B. Mandelbrot, J. Business **36**, 349 (1963).
5. J.-P. Bouchaud, D. Sornette, J. Phys. I France **4**, 863 (1994).
6. R.N. Mantegna, H.E. Stanley, Nature **376**, 46 (1995).
7. Y. Liu, P. Cizeau, M. Meyer, C.-K. Peng, H.E. Stanley, Physica A **245**, 437 (1997).
8. H. Takayasu, H. Miura, T. Hirabayashi, K. Hamada, Physica A **184**, 127 (1992); see also P. Bak, M. Paczuski, M. Shubik, Physica A **246**, 430 (1997).
9. H.E. Stanley, *Phase transitions and critical phenomena*, (Oxford Univ. Press, Oxford, 1971).
10. D. Sornette, Phys. Rep. **297**, 239 (1998).
11. J.C. Anifrani, C. Le Floc'h, D. Sornette, B. Souillard, J. Phys. I France **5**, 631 (1995).
12. A. Johansen, D. Sornette, H. Wakita, U. Tsunogai, W.I. Newman, H. Saleur, J. Phys. I France **6**, 1391 (1996).
13. D. Stauffer, D. Sornette, Physica A, in press (1998).
14. G. Sobotta, J. Phys. C **18**, 2065 (1985).
15. M.E. Fisher, Rev. Mod. Phys. **46**, 597 (1974).
16. J.M. Kosterlitz, D.J. Thouless, J. Phys. C **6**, 1181 (1973).
17. J.Toner, Yuhai Tu, Phys. Rev. Lett. **75**, 4326 (1995).
18. E. Brezin, J.C. LeGuillou, J. Zinn-Justin, Phys. Rev. Lett. **32**, 473 (1974).
19. L. Laloux, M. Potters, R. Cont, J.-P. Aguilar, J.-P. Bouchaud, cond-mat preprint 9804111.
20. D. Stauffer, private communication.
21. M. Ausloos, Europhys. News **29**, 70 (1998).
22. A. Johansen, D. Sornette, Eur. Phys. J. B **1**, 141 (1998).
23. A. Arnéodo, J.-F. Muzy, D. Sornette, Eur. Phys. J. B **2**, 277 (1998).
24. N. Vandewalle, Ph. Boveroux, A. Minguet, M. Ausloos, Physica A **255**, 201 (1998).